

Probing Deeper Mathematical Conceptual Understanding of PISA-like Uncertainty and Data

Wahyu Widada¹, Khathibul Umam Zaid Nugroho², Dewi Herawaty³

¹ Doctoral Program in Education, Bengkulu University, Bengkulu, Indonesia

² Program in Mathematics Education, PGRI University of Silampari, Lubuklinggau, Indonesia

³ Program in Mathematics Education, Bengkulu University, Bengkulu, Indonesia

¹ Corresponding Author: wahyu.unib@gmail.com

Abstrak

This study explores the effectiveness of "verbal probes" through in-depth interviews to uncover students' mathematical conceptual understandings that are not visible from written answers. Using a descriptive qualitative approach, this study analyzed the gap between written answers and verbal comprehension of three students with different skill levels on PISA-like questions themed Uncertainty and Data. The results showed a significant "gap between written answers and verbal comprehension". Students who give incorrect written answers, when interviewed, are able to articulate a deeper conceptual understanding, identify their mistakes, and explain correct procedures regarding the concepts of confidence intervals, exponential growth, and probability. In contrast, students with limited verbal comprehension consistently show reliance on number operations without a conceptual foundation. The study concludes that in-depth interviews are a crucial formative assessment tool to complement traditional methods. This allows educators to accurately diagnose misconceptions and design more holistic and targeted learning interventions.

Keywords: *conceptual comprehension, verbal probes, mathematical assessment, PISA-like, uncertainty and data.*

INTRODUCTION

A deep mathematical conceptual understanding is a crucial foundation for students' success in dealing with complex real-world challenges, especially in the digital age that increasingly relies on data and probabilistic reasoning (Aliev et al., 2024; Hernández-Solís et al., 2023; Lee et al., 2020). However, assessing conceptual understanding, especially in the context of mathematics, is often challenging. Written answer-based assessment results do not always reflect the true depth of student understanding, creating a "gap between written answers and verbal comprehension" (Hankeln, 2021). This phenomenon indicates that students may exhibit significantly different abilities when explaining concepts orally compared to what they put into writing. Previous research has highlighted that students' written answers can hide or even misrepresent their true level of conceptual

understanding (Rzyankina et al., 2024; Sinaga et al., 2022; Wittmann, 2020). One early indication of the limitations of written assessment is the tendency of students to perform "numbers operations without a deep conceptual understanding (Surface-Level Processing)" (Righteous, 2021). For example, some students tend to multiply or divide numbers haphazardly without a solid conceptual foundation (Kowiyah et al., 2019; Laily et al., 2020; Salmainsi et al., 2021; Setyawan, 2017; Widada et al., 2018). Even students who initially gave incorrect written answers and indicated an effort to simply process numbers, were then able to articulate better understanding verbally (Earl & Katz, 2006; Welji et al., 2022). This situation suggests that a purely written answer may reflect only surface strategy or even confusion, rather than an actual understanding of the concept.

Significant differences between written and verbal representations (Hwang & Hu, 2013; Septiani, 2020; Silmi Juman et al., 2022; Utami et al., 2019) is the core of the problem explored in this study. For example, students often give incorrect written answers, even similar to students whose understanding is very limited. However, "in interviews, they are able to articulate a much better understanding of concepts, identify errors in their written answers, and explain the approach they should take." Student 2, even though his written answer to Question 1 is incorrect, is able to explain the concept of the confidence level and the standard value of Z verbally. Similarly, Student 3 is also able to identify errors in his or her written answers and demonstrate a strong understanding of the concepts being tested when interviewed. This gap indicates that assessment methods that rely solely on written products risk providing an incomplete or even inaccurate picture of students' cognitive capabilities.

Uncovering this deeper understanding is important because conceptual understanding is the foundation for meaningful mathematics learning and knowledge transfer capabilities (Ahsan et al., 2024; Cahyono et al., 2020; Groth, 2018; Nirawati et al., 2021; Nurin et al., 2023; Plangg & Fuchs, 2022). If assessments fail to identify misconceptions or superficial understandings, appropriate learning interventions become difficult to design. Therefore, a method is needed that is able to transcend the limitations of written answers to dig deeper layers of understanding (subtle). "In-depth interviews managed to uncover significant differences in their conceptual understanding, problem-solving strategies, and metacognitive abilities", which may not have been detected through analysis of written answers alone.

This research contributes to the literature by exploring specifically the use of "verbal probes" in the context of PISA-like mathematical literacy questions with a focus on Uncertainty and Data content (Kong et al., 2022; Kusuma et al., 2022; Maslihah et al., 2020; Panich et al., 2024; Zeng, 2024). Although there have been many studies that discuss the importance of conceptual understanding (Cristia & Cueto, 2020; Dayal & Cowie, 2019; De Lange, 1999; Johansson, 2003; McFeetors et al., 2021; Sutton & Krueger, 2009; Widada et al., 2021), there are still limited studies that systematically investigate how verbal inquiry can effectively uncover the nuances of student understanding in specific domains relevant to the demands of the 21st century, such as data literacy and uncertainty (Cristia & Cueto, 2020). The need for data literacy is increasingly urgent amid the flood of information and

data in the digital era (Hikamudin et al., 2023; Ilham & Rahman, 2024; Weigand et al., 2024). The ability to understand, interpret, and critique data as well as probabilistic reasoning are key competencies tested within the PISA framework (OECD, 2019), but often students' deep understanding of these concepts is hidden behind their formal answers.

This research argues that a descriptive qualitative approach, particularly in-depth interviews, is the most appropriate method to capture the richness and complexity of student understanding that cannot be measured by standard written tests. By thematically analyzing data from in-depth interview transcriptions and comparing them with written answers, this study will provide strong empirical evidence of the value of "verbal probes" as a formative assessment tool that complements traditional methods. This approach not only identifies the correct or incorrect answers, but also delves into the reasons behind the students' responses, exposing misconceptions (Kadarisma et al., 2020; Luneta, 2015; Makonye & Fakude, 2016; Nguyena & Tran, 2014; Ozkan & Mehmet, 2012; Turmuzi et al., 2024), and highlight their metacognitive ability to manage information, even when information is missing in the question. This is in line with the view that assessments must be informative and able to facilitate learning, not just measurement (Dandis, 2013; van den Heuvel-Panhuizen et al., 2014).

Through a comparative analysis of written and verbal representations, this study aims to uncover how "verbal probes" should unveil a deeper and often different mathematical conceptual understanding of what is presented in a written format. The results of this study are expected to provide practical implications for educators in designing more comprehensive assessment strategies, enabling them to understand students' thinking processes more holistically and providing more targeted learning interventions, especially in the face of modern mathematical literacy challenges that prioritize data understanding and uncertainty (Miller, 2023; Rajendra et al., 2024). It will also strengthen the argument for the integration of qualitative methods in mathematical assessment research, paving the way for further study of the impact of verbal probes on students' development of long-term conceptual understanding.

Based on this background, this study aims to explore how the use of "verbal probes" through in-depth interviews can reveal students' deeper mathematical conceptual understanding, which is often not revealed through their written answers. In particular, this study focuses on PISA math literacy questions like (Andari & Setianingsih, 2021; Novitasari et al., 2021; Ortega-Rodríguez, 2025; Sepriyani, 2023; Siregar et al., 2020; Thomson et al., 2013; Xu et al., 2025) with Uncertainty and Data content, covering confidence intervals, exponential and logarithmic growth, and probability. This study will thematically analyze data from the transcription of in-depth interviews with three students, comparing their verbal comprehension with the written answers they have provided beforehand. Thus, this study is expected to contribute to the practice of assessment in mathematics education, highlighting the important value of dialogue and verbal interaction in an effort to understand students' thinking processes and depth of knowledge more comprehensively.

METHODS

This study uses a descriptive qualitative approach to explore in depth students' mathematical conceptual understanding, focusing on comparisons between written answers and verbally expressed understandings. This approach was chosen because it aims to uncover the nuances and depth of students' understanding that may not be captured through quantitative methods or written assessments alone.

Research Participants

The participants in this study were three students. The three students were selected by 1 person each from three groups of students: (1) Student Group with All Wrong Answers; (2) The group of students who answered correctly 25%-50%; and (3) the group of students who answered correctly at least 65%.

Instruments and Data Collection

Data collection is carried out through two main instruments:

1. **Written Test:** Students are given a series of math problems to work on in writing. This written answer is the initial data on student understanding before the interview is conducted. These questions are **PISA math literacy questions like with** Uncertainty and Data content, namely confidence intervals (**Question 1**), exponential growth and logarithm (**Question 2**), and probability (**Question 3**).
2. **Verbal Probes:** After students complete the written test, an in-depth interview is conducted individually with each student. This interview aims to:
 - Exploring students' conceptual understanding further regarding the answers they have written.
 - Provide opportunities for students to articulate their thought processes, identify errors in written answers, and explain the approach that should be.
 - Investigate aspects such as reliance on number operations without conceptual understanding, metacognitive abilities, identification and application of formulas, and sensitivity to information lost in problems. The interviews are recorded and then transcribed verbatim for analysis purposes.

Data Analysis

Data collected from the results of the written test and the transcription of in-depth interviews were analyzed using thematic analysis techniques. This analysis process involves the following steps:

1. **Data Familiarization:** Researchers repeatedly read interview transcripts and students' written answers to gain a thorough understanding.
2. **Initial Coding:** Identify relevant data segments from interview transcripts and written answers related to students' conceptual understanding, problem-solving strategies, and metacognitive processes.
3. **Theme Identification:** Grouping similar codes to form potential themes. The main focus is on themes that highlight the "gap between written answers and verbal comprehension", variations in depth of comprehension, and

students' ability to articulate comprehension that is not apparent in written answers.

4. **Theme Review:** Review and validate the themes that have been identified to ensure that they accurately represent the data and answer the research questions.
5. **Definition and Naming of Themes:** Provide clear definitions and representative names for each major theme that emerges from the analysis, as identified in the document (e.g., "Gap between Written Answers and Verbal Comprehension", "Reliance on Number Operations Without Deep Conceptual Understanding", etc.).

A comparative analysis was conducted between students' written answers and their verbal responses during the interview to uncover how "verbal probes" can "unveil" or reveal a deeper and often different mathematical conceptual understanding than what is presented in the written format.

RESULTS AND DISCUSSION

Data Reduction

Based on the transcription of the results of in-depth interviews on 3 research subjects (Student 1: Wrong Answer; Student 2: **Correct Answer 25-50%**; and Student 3: **Correct Answer 65% or more**). The transcription was reduced and presented in a question and answer format, summarizing the key points of the responses of each student from the three students

Student 1 (Wrong Answer):

Based on the results of paper-and-pencil work, and in-depth interviews, the Student 1 interview snippet is as follows.

Interviewer: Can you tell us how you understand this number 1 issue? What is asked?

Student 1: The problem is that it's like calculating the average temperature, and then there's a range. There is a number of -0.15°C equal to 95% confidence.

Interviewer: In your answer it says 'Average Temperature -0.15°C in 50 years'. Where did you get this -0.15°C number? Why are there negative signs?

Student 1: That's already in the question, sir/ma'am. Negative may mean that the temperature drops.

Interviewer: Then you write ' $0.15 \times 0130^{\circ}\text{C}$ '. Can you explain the meaning of this calculation and what is the number ' 0130°C '?

Student 1: Oh, that's... I think if there is a percentage or number like that, just multiply. The number 0.15 I took from the question, then 0130... Is that from the question too? I don't remember where it came from, but I think it was a few times.

Interviewer: What do you know about the '95% confidence level' and the 'standard deviation' in this matter? How are those two things supposed to be used?

Student 1: 95% confidence level is yes... 95% sure it is true. That standard deviation... Like the difference? I don't understand how to use it in this matter.

Interviewer: What steps did you have in mind to find the requested 'temperature range'?

Student 1: Yes, I tried to multiply the numbers. There may be other ways, but I don't know the formula.

The answers in Student 1's paper sheet are as follows (See Figure 1):

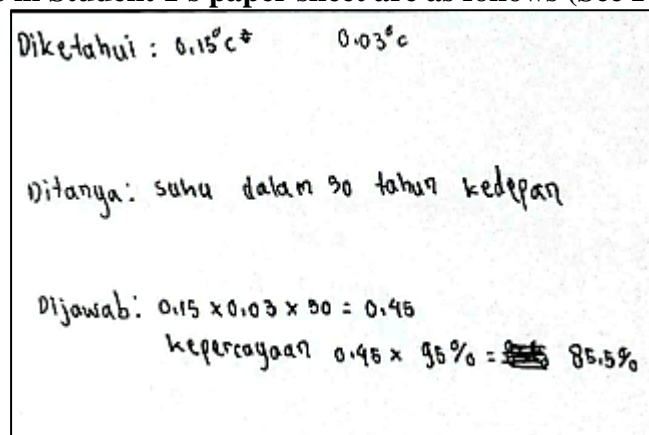


Figure 1 Student Answer 1 (Question 1)

Interviewer: What is the 2nd question about conservation?

Student 1: About the cases that increased by 10% every day, they kept asking when it became 10 times.

Interviewer: You wrote '10% : 10 times'. What is the relationship between 10% and 10 times in this matter?

Student 1: Yes, the question is 10% per day so that it becomes 10 times. So I wrote that.

Interviewer: Then there is the calculation '10/10 = 0 , 1 DL'. Can you explain how you came to this calculation? What do the numbers 10 above and below refer to? What does 'DLs' mean?

Student 1: That's what I'm trying to share. 10% is 0.1. And then you want to be 10 times. I divide 10 by 10, the result is 1. Then I wrote 0.1 again... I also forgot what 'DL' stands for, maybe 'double fold' or what? I just made it up at that time.

Interviewer: Did you think of using a specific formula for this kind of growth-related question?

Student 1: Formula? I didn't think of it, sir/ma'am. I don't know the formula.

Interviewer: How should we find the time it takes for cases to be 10 times if the growth per day is 10%?

Student 1: I don't know the exact steps. Maybe tried it out every day?

The answers in Student 1's paper sheet are as follows (See Figure 2):

diketahui: Pertumbuhan awal 10%

Ditanya:

Dijawab: $10\% = \frac{10}{100} \times 10^1 = 10$

7 hari = 1 minggu

$\frac{70}{100} \times 10 = \frac{700}{100} = 7$

Figure 2 Student Answer 1 (Question 2)

Interviewer: Can you explain what is asked in question number 3?

Student 1: Calculate the likelihood that two people have the same genotype. There is AA, Aa, aa.

Interviewer: You wrote ' $50 + 25 + 25 = 100 : 3$ '. Where did the numbers 50, 25, and 25 come from? Why add up and then divide by 3? What do you want to look for with this calculation?

Student 1: That's the percentage number of the problem. I add up all 100, and because there are 3 types of genotypes (AA, Aa, aa), I divide them by 3. I think it's average or something like that.

Interviewer: The result was '9.5% Genotype Aa'. How is this figure obtained? Is this the final answer to the probability of two individuals having the same genotype?

Student 1: 9.5% that... The result of dividing 100 by 3 is about 33 commas. I don't know where to get 9.5% from. It seems to be a miscalculation or a complete misconception. No, that's not the final answer to the probability of two individuals having the same genotype.

Interviewer: If we want to look for the probability of two random events occurring simultaneously (e.g., the first individual of AA and the second individual of AA), what mathematical operation is usually used?

Student 1: If the "and" is... Plus? Or reduced? I forgot the lesson of probability.

Interviewer: How do the correct steps signal to solve this problem?

Student 1: I don't know the right move, sir/ma'am. I was confused about where to start.

The answers in Student 1's paper sheet are as follows (See Figure 3):

Diketahui: $AA = 25\%$ $50\% Aa$
 $aa = 25\%$

Ditanya:

Dijawab: $AA \times Aa \times aa$
 $= 3A \times 3a$
 $= 3:3$

Figure 3 Student Answer 1 (Question 3)

Student 2 (Correct Answer 25-50%)

Interviewer: Can you tell us how you understand this number 1 issue? What is asked?

Student 2: This is about the change in average temperature over 50 years, which is a decrease of 0.15C. They are constantly asked to find the temperature range with a 95% confidence level. That is, the range in which the true average temperature is estimated to be with 95% confidence.

Interviewer: In your answer it says 'Average Temperature -0.15C in 50 years'. Where did you get this -0.15C number? Why are there negative signs?

Student 2: This number is already in the question. A negative sign means that the temperature has decreased on average.

Interviewer: Then you write '0.15 x 0130°C'. Can you explain the meaning of this calculation and what is the number '0130°C'?

Student 2: Oh, this... This answer seems to be wrong, sir/ma'am. This calculation of 0.15 x 0130°C is baseless. I also don't remember where the number 0130°C came from, maybe I misunderstood another problem or thought it was a standard deviation, but it was clearly wrong.

Interviewer: What do you know about the '95% confidence level' and the 'standard deviation' in this matter? How are those two things supposed to be used?

Student 2: The 95% confidence level means that there is a 5% chance that the data we have is outside that range. To find a range at a given confidence level, we need the mean, the standard deviation, and the Z-value of the

standard normal table. For 95%, the Z-value is around 1.96. The formula is an average plus and minus (the value of Z is multiplied by the standard deviation divided by the root n). But in this case the standard deviation is not given, so I am confused as to how to calculate it.

Interviewer: What steps did you have in mind to find the requested 'temperature range'?

Student 2: The steps should be using the confidence interval formula: identify the mean (-0.15C), find a Z-value for 95% (1.96), need a standard deviation and sample size (50 years, maybe this is n?), calculate the lower and upper limits. But since the standard deviation is not there, I can't continue.

The answers in Student 2's paper sheet are as follows (See Figure 4):

Diketahui : rata-rata = 0,15
Standar Deviasi = 0,03

Ditanya : Peningkatan suhu rata-rata dalam 5 Dekade, $\alpha = 95\%$

Jawab :

Peningkatannya adalah :

$$5 \times 0,15 = 0,75$$

Jadi $0,75^{\circ}\text{C}$.

Figure 4 Student Answer 2 (Question 1)

Interviewer: What is the 2nd question about conservation?

Student 2: It's about growth, exponential growth of cases by 10% every day. He was asked how long it would take for the number of cases to be 10 times the initial number.

Interviewer: You wrote '10%: 10 times'. What is the relationship between 10% and 10 times in this matter?

Student 2: 10% is the growth rate every day. 10 times that target number of final cases compared to the initial number.

Interviewer: Then there is the calculation '10/10 = 0,1 DL'. Can you explain how you came to this calculation? What do the numbers 10 above and below refer to? What does 'DLs' mean?

Student 2: This calculation is also wrong, sir/ma'am. I try to associate the number 10%, maybe I think 10% is 10/100, then I want to be 10 times so there is another number 10. But $10/10 = 1$, not 0.1. I don't remember what 'DL' was abbreviated, maybe I just composed it. This is clearly a

misconception.

Interviewer: Did you think of using a specific formula for this kind of growth-related question?

Student 2: Yes, you should use the exponential growth formula:
 $N(t) = N_0(1+r)^t$.

Interviewer: How should we find the time it takes for cases to be 10 times if the growth per day is 10%?

Student 2: We want $N(t) = 10 * N_0$. So, $10 * N_0 = N_0 * (1+0.10)^t$. $10 = (1.10)^t$. To find t , we need to use a logarithm: $t = \log(10)/\log(1.10)$.

The answers in Student 2's paper sheet are as follows (See Figure 5):

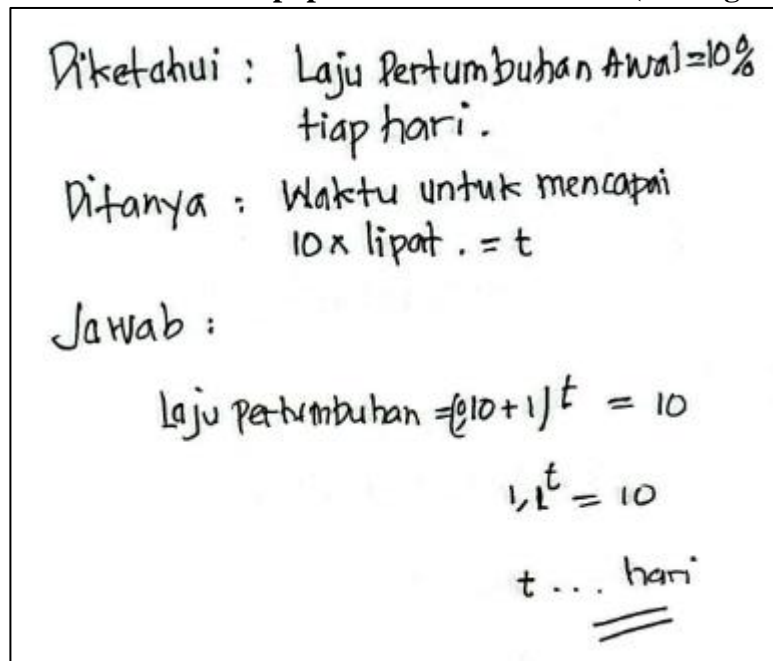


Figure 5 Student Answer 2 (Question 2)

Interviewer: Can you explain what is asked in question number 3?

Student 2: Asked if we take two individuals randomly, they both have exactly the same genotype.

Interviewer: You wrote ' $50 + 25 + 25 = 100 : 3$ '. Where did the numbers 50, 25, and 25 come from? Why add up and then divide by 3? What do you want to look for with this calculation?

Student 2: The numbers 50, 25, and 25 are the probabilities of each genotype given in the question. I summed it up because... I think this has something to do with total probability. But why divide it by 3? Oh, that's wrong, sir/ma'am. I shouldn't have divided 3.

Interviewer: The result was '9.5% Genotype Aa'. How is this figure obtained? Is this the final answer to the probability of two individuals having the same genotype?

Student 2: This 9.5% figure... This is the wrong result of the wrong calculation. This is not the correct final answer.

Interviewer: If we want to look for the probability of two random events

occurring simultaneously (e.g., the first individual of AA and the second individual of AA), what mathematical operation is usually used?

Student 2: If there is a word "and" for events that are independent of each other, we use probability multiplication. So, the probability of the first individual of AA and the second individual of AA is the probability of AA multiplied by the probability of AA.

Interviewer: How do the correct steps signal to solve this problem?

Student 2: Correct step: First, identify the possible genotypes of the same: (AA and AA), (Aa and Aa), or (aa and aa). Second, calculate the probability for each of these possibilities by multiplying the probability of each. $P(\text{AA and AA}) = 0.25 \times 0.25 = 0.0625$. $P(\text{Aa and Aa}) = 0.50 \times 0.50 = 0.25$. $P(\text{aa and aa}) = 0.25 \times 0.25 = 0.0625$. Third, since these three possibilities are mutually exclusive, we add up the probabilities: $0.0625 + 0.25 + 0.0625 = 0.375$. So, the probability is 37.5%.

The answers in Student 2's paper sheet are as follows (See Figure 6):

Diketahui: AA = 25%
Aa = 50%
aa = 25%

Ditanya: Peluang sama?

Jawab:

$$P(\text{sama}) = P(\text{AA}) + P(\text{Aa}) + P(\text{aa})$$

$$= 25\% + 50\% + 25\%$$

$$= 100\%$$

Figure 6 Student Answer 2 (Question 3)

Student 3 (Correct Answer Minimum 65%)

Interviewer: Can you tell us how you understand this number 1 issue? What is asked?

Student 3: This question provides information about the change in average temperature over 50 years, i.e. there is an average decrease of 0.15

degrees Celsius. Then asked the temperature range, which seems to refer to the confidence interval for the average temperature change at a 95% confidence level.

Interviewer: In your answer it says 'Average Temperature -0.15C in 50 years'. Where did you get this -0.15C number? Why are there negative signs?

Student 3: This -0.15C number is directly given in the question as the average temperature change. A negative sign indicates that there is a drop in temperature.

Interviewer: Then you write '0.15 x 0130°C'. Can you explain the meaning of this calculation and what is the number '0130°C'?

Student 3: Oh, this is my written answer seems to be wrong, sir/ma'am. The calculation of 0.15 times 0130 is irrelevant to finding the confidence interval. I don't remember where the number 0130 came from. This is not the right move.

Interviewer: What do you know about the '95% confidence level' and the 'standard deviation' in this matter? How are those two things supposed to be used?

Student 3: The 95% confidence level means that if we sample many times and calculate the confidence intervals, 95% of those intervals will contain the actual population parameter values. Standard deviation measures the distribution of data. We need the standard deviation of the sample to calculate the standard error average, which is used in conjunction with the value of Z (1.96) to establish the confidence interval. The formula is approximately an average of $\pm (Z/t \text{ value} \times \text{standard error})$. Standard error is usually calculated from the standard deviation divided by the root of the sample size.

Interviewer: What steps did you have in mind to find the requested 'temperature range'?

Student 3: First, identify the sample mean (-0.15C) and sample size ($n=50$ years). Second, determine the level of trust (95%). Third, look for the critical value of Z (1.96). Fourth, we need the standard deviation value of this temperature change, which is not given in the problem. If the standard deviation of the sample(s) is known, we calculate the standard error: $SE = s / \sqrt{n}$. Fifth, calculate the confidence interval: Average $\pm (Z \times SE)$. Since the standard deviation does not exist, this problem cannot be calculated numerically.

The answers in Student 3's paper sheet are as follows (See Figure 7):

Diket: $\bar{x} = 0,15$
 $sd = 0,03$
 Dit: $\alpha = 95\%$
 $t = 50 \text{ tahun} = 5 \text{ dekade}$
 Jawab:
 Pakai angka baku $z = 95\%$
 adalah 1,96
 maka peningkatan suhu rata-rata
 dalam 5 dekade
 $0,15 \times 5 = 0,75$
 yaitu 95°C .

Figure 7 Student Answer 3 (Question 1)

Interviewer: What is the 2nd question about conservation?

Student 3: This question is about the exponential growth model. The daily growth rate is known and it takes time to reach a certain multiple of the initial value.

Interviewer: You wrote '10%: 10 times'. What is the relationship between 10% and 10 times in this matter?

Student 3: 10% is the relative growth rate per period of time (per day). 10 times is the multiplier factor to be achieved from the initial amount.

Interviewer: Then there is the calculation '10/10 = 0, 1 DL'. Can you explain how you came to this calculation? What do the numbers 10 above and below refer to? What does 'DLs' mean?

Student 3: This calculation is a wrong scribble, sir/ma'am. This is not the right way to solve the problem of exponential growth. I don't remember the meaning of this 'DL'.

Interviewer: Did you think of using a specific formula for this kind of growth-related question?

Student 3: Yes, this problem is solved using the exponential growth formula: $N(t) = N_0 * (1+r)^t$.

Interviewer: How should we find the time it takes for cases to be 10 times if the growth per day is 10%?

Student 3: We want to find t when $N(t) = 10 * N_0$. So, $10 * N_0 = N_0 * (1+0.10)^t$. $10 = (1.10)^t$. To find t , we use logarithms: $t = \log(10) / \log(1.10)$. Using a calculator, $\log(10) = 1$ and $\log(1.10) = 0.0414$. So $t = 1 / 0.0414 = 24.39$ days.

The answers in Student 3's paper sheet are as follows (See Figure 8):

Diket: $L(t) = 10\% / \text{hari}$.

Dit: t mencapai 10 kali lipat.

Jawab:

$$10 = L(t) = 1 \cdot (1 + 10\%)^t$$

$$= (1 + 0,1)^t$$

$$10 = 1,1^t$$

$$\Rightarrow t = ?$$

$$\log 10 = \log (1,1)^t$$

$$\Rightarrow t \log 1,1 = \log 10$$

$$\Rightarrow t = \frac{1}{\log 1,1}$$

$$\Rightarrow t = \frac{1}{0,091} \quad (\text{pakai kalkulator})$$

$$\Rightarrow t = 24,39 \text{ hari}$$

=

Figure 8 Student Answer 3 (Question 2)

Interviewer: Can you explain what is asked in question number 3?

Student 3: Asked the probability that two individuals taken independently of this population would have identical genotypes, i.e. AA and AA, or Aa and Aa, or aa and aa.

Interviewer: You wrote ' $50 + 25 + 25 = 100 : 3$ '. Where did the numbers 50, 25, and 25 come from? Why add up and then divide by 3? What do you want to look for with this calculation?

Student 3: The numbers 50, 25, and 25 are the probabilities of each genotype. The sum of $50+25+25=100\%$ is true because it is the total probability. But dividing it by 3 is wrong, sir/ma'am. Maybe try to find the average, but it's irrelevant.

Interviewer: The result was '9.5% Genotype Aa'. How is this figure obtained? Is this the final answer to the probability of two individuals having the same genotype? **Student 3:** This 9.5% figure is the result of an incorrect calculation, so it's not the correct answer.

Interviewer: If we want to look for the probability of two random events occurring simultaneously (e.g., the first individual of AA and the second individual of AA), what mathematical operation is usually used?

Student 3: If the two events are independent, to find the probability of both occurring ("and"), we multiply the probability of each event.

Interviewer: How do the correct steps signal to solve this problem? **Student**

3: Step: First, determine all possible pairs of the same genotype: (AA, AA), (Aa, Aa), and (aa, aa). Second, calculate the probability for each pair using the multiplication rule: $P(\text{AA and AA}) = 0.25 \times 0.25 = 0.0625$; $P(\text{Aa and Aa}) = 0.50 \times 0.50 = 0.25$; $P(\text{aa and aa}) = 0.25 \times 0.25 = 0.0625$. Third, since these pairs are separate events, add up the probabilities: $P(\text{Same Genotype}) = 0.0625 + 0.25 + 0.0625 = 0.375$. So, the probability is 37.5%.

The answers in Student 3's paper sheet are as follows (See Figure 9):

Diket: $gen\ AA = 0,25$
 $gen\ Aa = 0,5$
 $gen\ aa = 0,25$

 Dit.: Prob. dua individu yg dipilih acak bergenotif sama.

 Jawab:
 Pilih ser. acak dua individu; maka terpilih:
 $AA = 0,25 \times 0,25 = 0,0625$
 $Aa = 0,5 \times 0,5 = 0,25$
 $aa = 0,25 \times 0,25 = 0,0625$

 $P(\text{sama}) = P(AA, AA) + P(Aa, Aa) + P(aa, aa)$
 $= 0,0625 + 0,25 + 0,0625$
 $P(\text{sama}) = 0,375$

Figure 9 Student Answer 3 (Question 3)

Thematic Analysis

Based on the results of the in-depth interview data with the three students, several main themes can be identified through thematic analysis techniques:

1. Gap between Written Answers and Verbal Comprehension

This theme highlights the significant differences between what students write as answers and the conceptual understanding they verbally demonstrate during the interview.

- **Student 1:** Demonstrated consistency between incorrect written answers and very limited verbal comprehension in all questions.
- **Student 2 and Student 3:** Both often give incorrect written answers, even similar to Student 1's answer (for example, Question 1 says '0.15 x 0130°c' and Question 2 '10/10 = 0.1 DL'). However, in interviews, they are able to articulate a much better understanding of concepts, identify errors in their written answers, and explain the approach they should take.
 - **Student 2:** Even if the written answer to Question 1 is wrong, he can explain the concepts of confidence level, the need for standard deviation, and the value of Z. For Question 2, he recognizes the exponential growth formula and the use of logarithms. For Question 3, he correctly explained how to calculate the combined probability.
 - **Student 3:** Also identify his or her written answer errors for all three questions. He demonstrated a strong understanding of confidence intervals (Question 1), exponential growth and logarithms (Question 2), and probability calculations (Question 3).

2. Reliance on Number Operations Without Deep Conceptual Understanding (Surface-Level Processing)

This theme describes the tendency of students to perform mathematical operations on the numbers available in the question without a strong understanding of the concepts being tested.

- **Student 1:** Consistently exhibits this pattern. For example, he multiplies the numbers in Question 1 because "if there is a percentage or number like that, just multiply" and divides the numbers in Question 2 arbitrarily. In Question 3, he adds up all the percentages and then divides them by the number of genotype categories, thinking it's to find an average.
- **Student 2 and Student 3:** Their initial written answers to Question 1 ('0.15 x 0130°c') and Question 2 ('10/10 = 0, 1 DL') indicate an initial attempt to simply process the given number, before they realized and corrected the approach verbally.

3. Variations on the Depth of Understanding of Scientific and Mathematical Concepts

Interviews reveal differences in the level of understanding of concepts between students:

- **Student 1:** Very superficial understanding or misconception of almost all concepts tested, such as confidence levels, standard deviations, exponential growth, and the basics of probability.

- **Student 2:** Demonstrate a developing understanding. He was able to identify key concepts such as confidence intervals and Z-values (Question 1), exponential growth formulas and logarithms (Question 2), and rules of multiplication and addition in probability (Question 3). However, he is sometimes still hesitant or hampered by a lack of information (e.g., the standard deviation in Question 1).
- **Student 3:** Demonstrate the most powerful and profound conceptual understanding. He understands not only formulas and procedures but also the assumptions and limitations that underlie them. For example, he explains the meaning of the 95% confidence level, the importance of standard deviations and how standard errors are calculated (Question 1), the application of logarithms in exponential growth with estimation calculations (Question 2), and systematic steps in solving complex probability problems (Question 3).

4. Metacognitive Abilities and Fault Recognition

This theme relates to students' awareness of their own thought processes and the ability to identify mistakes.

- **Student 1:** Often admits "don't know", "forget", or "ngarang". This shows an awareness of its limitations, even though it has not been able to overcome them.
- **Student 2 and Student 3:** Both actively identify and admit errors in their written answers during the interview. They can explain why their initial approach was wrong, which indicates a higher level of metacognition.

5. Identification and Application of Mathematical Formulas/Procedures

The ability of students to remember and apply the right formula or procedure varies.

- **Student 1:** Explicitly states that they did not think of using the formula or did not know the formula for Questions 1 and 2. For Question 3, he also did not know the correct procedure.
- **Student 2:** Able to name the correct formula for exponential growth (Question 2) and explain the components of the confidence interval formula (Question 1) and the rule of probability (Question 3).
- **Student 3:** Not only remember the relevant formulas (confidence interval, exponential growth) but also explain how to use them in detail and accurately, including the use of logarithms and the steps of calculating probabilities.

6. Sensitivity to Information Lost in Questions

This theme especially appears in Question 1, where information about standard

deviations is not included.

- **Student 1:** Unaware of missing information or the importance of standard deviations.
- **Student 2 and Student 3:** Both independently identified that the standard deviation was not given in the question, and this precludes the numerical calculation for the temperature range (confidence interval). It shows a deeper understanding of the prerequisites and limitations of the application of statistical methods.

This thematic analysis showed that although all three students may have produced similar written answers in some cases, in-depth interviews managed to uncover significant differences in their conceptual understanding, problem-solving strategies, and metacognitive abilities.

Discussion

This discussion focused on the results of data reduction from in-depth interviews with three students, highlighting differences between verbally expressed conceptual understandings compared to their written answers. Student 1, who gave the overall incorrect written answer, showed very limited conceptual understanding during the interview. He tends to perform "numbers operations without a deep conceptual understanding (Surface-Level Processing)", such as multiplying numbers without a clear basis or dividing carelessly. Student 1 is also unaware of the use of relevant formulas for exponential growth or probability problems. This is consistent with research showing that written assessments alone often fail to capture a lack of deep understanding (Nasution & Hafizah, 2020; Purba & Surya, 2020; Swaroop et al., 2024; Varma et al., 2021).

In contrast, Student 2 and Student 3, despite initially giving erroneous written answers to some passages, showed a much better ability at verbally articulating their conceptual understanding. They were able to identify errors in their written answers and explain the approach they should take, including mentioning relevant formulas such as confidence intervals and exponential growth as well as probability multiplication rules for independent events. These findings support the argument that "verbal probes" can uncover deeper understandings that are not detected through written assessments alone, highlighting the "gap between written answers and verbal comprehension" that is crucial for effective learning interventions (De La Paz & van Garderen, 2024; Faber et al., 2024; Zakariya, 2023). Their ability to revise comprehension when interviewed suggests the potential for comprehension that has not yet been fully manifested in written form, in line with the study of metacognition in mathematical problem-solving (Wijn et al., 2023; Barwasser et al., 2024; Hsiao et al., 2018; Jie et al., 2022; Le et al., 2024).

CONCLUSION

The conclusion of this study confirms that "verbal probes" through in-depth

interviews are effective in revealing students' mathematical conceptual understanding, especially in PISA-like Uncertainty and Data questions. Written answers often do not reflect the depth of comprehension, creating a "gap between written answers and verbal comprehension". Student 2 and Student 3, even if the written answers are wrong, are able to articulate a better understanding of concepts verbally, identify mistakes, and explain the approach that should be. They demonstrate an understanding of confidence intervals, exponential growth, and probability. In contrast, Student 1 whose entire answer is wrong, shows reliance on number operations without a deep understanding and difficulty in identifying relevant formulas. In-depth interviews managed to dig into more subtle layers of understanding that were not detected through written assessments.

Suggestions for teachers to integrate "verbal probes" in assessments for early identification of misconceptions and students' deep understanding.

The implication of this study is the need to shift the assessment paradigm to holistic assessment that includes students' thinking processes for more targeted learning interventions.

Advanced Research is the expansion of the sample, the exploration of different types of verbal probes, the comparative study of the impact of interventions based on "verbal probes", and the development of students' metacognitive related to comprehension gaps.

REFERENCE

- Ahsan, M. G. K., Cahyono, A. N., & Kharisudin, I. (2024). Designing digital math trail environment assisted by augmented reality using mathematical modeling learning approach. In H. H. & R. T. (Eds.), *AIP Conference Proceedings* (Vol. 3106, Issue 1). American Institute of Physics. <https://doi.org/10.1063/5.0215762>
- Aliev, R. A., Kacprzyk, J., Pedrycz, W., Mo. Jamshidi, M. B. B., & Sadikoglu, F. M. (2024). 16th International Conference on Application of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools, ICAFS 2023. *Lecture Notes in Networks and Systems*, 1141 LNNS <https://www.scopus.com/inward/record.uri?eid=2-s2.0-85210264139&partnerID=40&md5=6aad51a8e2c277c9adcff33fd5a7fc>
- Andari, R. M., & Setianingsih, R. (2021). Students' Mathematical Literacy in Solving PISA Problem Using Indonesian Cultural Context. *6*(1), 52–67.
- Anggoro, A. F. D., Hendriana, H., & Yuliani, A. (2023). Structural Model Between Mathematical Reasoning and Mathematics Problem-Solving Abilities of Junior High School Students. *Journal of Applied Mathematics*, *6*(April), 71–75. <http://www.journalfkipunipa.org/index.php/jhm/article/view/405%0Ahttps://www.journalfkipunipa.org/index.php/jhm/article/download/405/241>
- Barwasser, A., Schulze, S., Gieseler, C., & Grünke, M. (2024). Effects of a Math Single-Case Intervention on Word Problem-Solving in Students With

- Learning Disabilities and Emotional and Behavioral Disorders. *Exceptional Children*. <https://doi.org/10.1177/00144029241247037>
- Cahyono, A. N., Sukestiyarno, Y. L., Asikin, M., Ahsan, M. G. K., & Ludwig, M. (2020). Learning mathematical modelling with augmented reality mobile math trails program: How can it work? *Journal on Mathematics Education*, 11(2), 181–192. <https://doi.org/10.22342/jme.11.2.10729.181-192>
- Cristia, J., & Cueto, S. (2020). Learning Mathematics in the 21 st Century EDITORS Elena Arias Ortiz.
- Dandis, M. A. (2013). The assessment methods that are used in a secondary mathematics class. *Journal of Educators, Teachers and Trainers*, 4(2), 133–143. [https://www.ugr.es/~jett/pdf/vol04\(2\)_Maha_Dandis.pdf](https://www.ugr.es/~jett/pdf/vol04(2)_Maha_Dandis.pdf)
- Dayal, H. C., & Cowie, B. (2019). Professional learning intervention in mathematics: A case of developing portfolio assessment. *Australian Journal of Teacher Education*, 44(2). <https://doi.org/10.14221/ajte.2018v44n2.7>
- De La Paz, S., & van Garderen, D. (2024). Current Research Informing the Conceptualization of STEM Interventions for Students With LD: An Introduction to the Special Series. *Learning Disability Quarterly*, 47(2), 63–69. <https://doi.org/10.1177/07319487241235144>
- De Lange, J. (1999). Framework for Classroom Assessment in Mathematics. In National Center for Improving Student Learning and Achievement in Mathematics and Science (issue September). Freudenthal Institute.
- Earl, L., & Katz, S. (2006). Rethinking Classroom Assessment with Purpose in Mind. In Learning. www.wncp.ca
- Faber, T. J. E., Dankbaar, M. E. W., van den Broek, W. W., Bruinink, L. J., Hogeveen, M., & van Merriënboer, J. J. G. (2024). Effects of adaptive scaffolding on performance, cognitive load and engagement in game-based learning: a randomized controlled trial. *BMC Medical Education*, 24(1). <https://doi.org/10.1186/s12909-024-05698-3>
- Groth, R. E. (2018). Introduction to Teaching and Learning Mathematics. <https://doi.org/10.4135/9781452244105>
- Hankeln, C. (2021). Combining and Contrasting Formal Concept Analysis and APOS Theory. *Educational Studies in Mathematics*, 105(1), 1–14. <https://doi.org/10.1007/s10649-018-9859-7>
- Hernández-Solís, L. A., Batanero, C., & Gea, M. M. (2023). Costa Rican students' proportional reasoning and comparing probabilities in spinners. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(12). <https://doi.org/10.29333/ejmste/13869>
- Hikamudin, E., Riyadi, A. R., Aryanti, Peniasiani, D., Nuryani, P., & Gofur, R. (2023). Improving Elementary School Students' Understanding of Literacy and Numeracy Through Digital Applications. *MIMBAR PGSD Undiksha*, 11(3), 462–467. <https://doi.org/10.23887/jjgsd.v11i3.64018>

- Hsiao, H. S., Lin, C. Y., Chen, J. C., & Peng, Y. F. (2018). The influence of a mathematics problem-solving training system on first-year middle school students. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(1), 77–93. <https://doi.org/10.12973/ejmste/77902>
- Hwang, W.-Y., & Hu, S.-S. (2013). Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. *Computers and Education*, 62, 308–319. <https://doi.org/10.1016/j.compedu.2012.10.005>
- Ilham, M., & Rahman, F. (2024). Character Education of Local Wisdom-based: A Study of Moral Aspect of Quotes belong to Bugis People. *Journal of Ecohumanism*, 3(3), 1125–1140. <https://doi.org/10.62754/joe.v3i3.3443>
- Jie, Z., Li, J., & Lu, W. (2022). Learning to Reason Deductively: Math Word Problem Solving as Complex Relation Extraction. In <https://github.com/allanj/Deductive-MWP>. <http://arxiv.org/abs/2203.10316>
- Johansson, M. (2003). Textbooks in mathematics education: A study of textbooks as the potentially implemented curriculum. Thesis, 88. urn:nbn:se:ltu:diva-18457
- Kadarisma, G., Fitriani, N., & Amelia, R. (2020). Relationship Between Misconception and Mathematical Abstraction of Geometry At Junior High School. *Infinity Journal*, 9(2), 213–222. <https://doi.org/10.22460/infinity.v9i2.p213-222>
- Kong, S.-C., Cheung, W. M.-Y., & Zhang, G. (2022). Evaluating artificial intelligence literacy courses for fostering conceptual learning, literacy and empowerment in university students: Refocusing to conceptual building. *Computers in Human Behavior Reports*, 7. <https://doi.org/10.1016/j.chbr.2022.100223>
- Kowiyah, K., Mulyawati, I., & Umam, K. (2019). Conceptual Understanding and Mathematical Representation Analysis of Realistic Mathematics Education Based on Personality Types. *Al-Jabar : Journal of Mathematics Education*, 10(2), 201–210. <https://doi.org/10.24042/ajpm.v10i2.4605>
- Kusuma, D., Sukestiyarno, Y. L., & Cahyono, A. N. (2022). The characteristics of mathematical literacy based on Students' executive function. *European Journal of Educational Research*, 11(1), 193–206. <https://doi.org/10.12973/eu-er.11.1.193>
- Laily, N. I., Yuliati, N., Imamah, E. N., & Sari, M. P. (2020). The profile of students conceptual understanding and procedural knowledge in solving geometry problems based on van hiele levels. In D. null (Ed.), *Journal of Physics: Conference Series* (Vol. 1538, Issue 1). Institute of Physics Publishing. <https://doi.org/10.1088/1742-6596/1538/1/012080>
- Le, N.-K., Nguyen, D.-H., Do, D.-T., Nguyen, C., & Nguyen, M. L. (2024). Vietnamese Elementary Math Reasoning Using Large Language Model with Refined Translation and Dense-Retrieved Chain-of-Thought. In S. T. & B. M.

- (Eds.), *Lecture Notes in Computer Science* (including subseries *Lecture Notes in Artificial Intelligence* and *Lecture Notes in Bioinformatics*): Vol. 14741 LNAI (pp. 260–268). Springer Science and Business Media Deutschland GmbH. https://doi.org/10.1007/978-981-97-3076-6_18
- Lee, W., Yu, H., Rival, X., & Yang, H. (2020). Towards verified stochastic variational inference for probabilistic programs. *Proceedings of the ACM on Programming Languages*, 4(POPL). <https://doi.org/10.1145/3371084>
- Luneta, K. (2015). Understanding students' misconceptions: An analysis of final grade 12 examination questions in geometry. *Pythagoras*, 36(1), 1–11. <https://doi.org/10.4102/pythagoras.v36i1.261>
- Makonye, J. P., & Fakude, J. (2016). A Study of Errors and Misconceptions in the Learning of Addition and Subtraction of Directed Numbers in Grade 8. *SAGE Open*, 6(4). <https://doi.org/10.1177/2158244016671375>
- Maslihah, S., Waluya, S. B., Rochmad, & Suyitno, A. (2020). The Role of Mathematical Literacy to Improve High Order Thinking Skills. *Journal of Physics: Conference Series*, 1539(1). <https://doi.org/10.1088/1742-6596/1539/1/012085>
- McFeetors, J., Marynowski, R., & Candler, A. (2021). Generative unit assessment: Authenticity in mathematics classroom assessment practices. *Education Sciences*, 11(7). <https://doi.org/10.3390/educsci11070366>
- Miller, D. D. (2023). The calculated uncertainty of scientific discovery: From Maths to Deep Maths. In K. S.G., S. L. Washington University in St. Louis MO, S. R. A.S.R., A. Medical College of Georgia GA, R. C.R., & U. of H. C. AIMSCS Hyderabad (Eds.), *Handbook of Statistics* (Vol. 49, pp. 203–226). Elsevier B.V. <https://doi.org/10.1016/bs.host.2023.05.001>
- Nasution, M. L., & Hafizah, N. (2020). Development of students' understanding of mathematical concept with STAD type cooperative learning through student worksheets. *Journal of Physics: Conference Series*, 1554(1). <https://doi.org/10.1088/1742-6596/1554/1/012035>
- Nguyena, G.-N. T., & Tran, D. (2014). Student Understanding of Function Concepts in K-16 : Consistency and Misconceptions. *RMERA Conference 2014 Research Paper*, <https://www.researchgate.net/publication/276026804> Student, 1–10.
- Nirawati, R., Fatimah, S., & Juandi, D. (2021). Realistic Mathematics Learning on Students' Ways of Thinking. *Mathematics Teaching-Research Journal*, 13(4), 112–130. <https://www.scopus.com/inward/record.uri?eid=2-s2.0-85127186264&partnerID=40&md5=ebb7796fd80317310b224d79911d2211>
- Novitasari, D., Nasrullah, A., Wahyu Triutami, T., Ayu Apsari, R., & Silviana, D. (2021). High level of visual-spatial intelligence's students in solving PISA geometry problems. In J. A., Y. W. N., U. R., & P. P. W. (Eds.), *Journal of Physics: Conference Series* (Vol. 1778, Issue 1). IOP Publishing Ltd. <https://doi.org/10.1088/1742-6596/1778/1/012003>

- Nurin, N. S., Ahsan, M. G. K., & Cahyono, A. N. (2023). Learning mathematical modeling in a virtual mobile math trails environment. In S. null, S. K., C. E., M. null, L. C.P., & A. S. (Eds.), *AIP Conference Proceedings* (Vol. 2614). American Institute of Physics Inc. <https://doi.org/10.1063/5.0125858>
- Ortega-Rodríguez, P.-J. (2025). Pisa 2022: The impact of school-environment predictors on the performance of Spanish students. *Spanish Journal of Pedagogy*, 83(290), 223–240. <https://doi.org/10.22550/2174-0909.4100>
- Ozkan, A., & Mehmet, E. (2012). Misconceptions and Learning Difficulties in Radical Numbers. *Social and Behavioral Sciences*, 46, 462–467. <https://doi.org/10.1016/j.sbspro.2012.05.142>
- Panich, S., Yasri, P., & Toopsuwan, C. (2024). Development of Game-Based Learning on Personal Income Tax Literacy and Mathematical Decision Making for Strategic Tax Planning. 9th International STEM Education Conference, ISTEM-Ed 2024 - Proceedings. <https://doi.org/10.1109/iSTEM-Ed62750.2024.10663196>
- Plangg, S., & Fuchs, K. J. (2022). A Gender-Related Analysis of a Robots' Math Class. *International Journal for Technology in Mathematics Education*, 29(3), 143–163. https://doi.org/10.1564/tme_v29.3.03
- Purba, G. I. D., & Surya, E. (2020). The Improving of Mathematical Understanding Ability and Positive Attitudes of Unimed FMIPA Students by Using the Contextual Teaching Learning (CTL) Approach. In R. M., S. H., & R. J. (Eds.), *Journal of Physics: Conference Series* (Vol. 1462, Issue 1). Institute of Physics Publishing. <https://doi.org/10.1088/1742-6596/1462/1/012019>
- Rajendra, P., Ravi, P. V. N. H., & Meenakshi, K. (2024). Machine learning from a mathematical perspective. In S. M.S., S. B., & B. S. (Eds.), *AIP Conference Proceedings* (Vol. 3149, Issue 1). American Institute of Physics. <https://doi.org/10.1063/5.0225046>
- Rzyankina, E., George, F., & Simpson, Z. (2024). Enhancing Conceptual Understanding in Engineering Mathematics Through E-Textbooks. *IEEE Transactions on Education*, 67(4), 534–541. <https://doi.org/10.1109/TE.2024.3387102>
- Salmainsi, S., Fauzan, A., & Arnawa, I. M. (2021). The Relationship Between the Ability of Understanding Mathematical Concepts With Language Ability During Contextual Learning. *Advances in Social Science, Education and Humanities Research*, 532(ICETeP), 428–433.
- Sepriliani, S. P. (2023). Development of PISA-like Activities using the Inquiry-based Learning Model and the Context of Religious Holidays during the Pandemic. *Mathematics Education Journal*, 17(1), 37–54. <https://doi.org/10.22342/jpm.17.1.17765.37-54>
- Septiani, D. (2020). Students' Verbal Representation Abilities in Solving Geometry Problems. In R. null, I. I., Y. Y., K. null, A. Z., & R. R. (Eds.), *Journal of Physics: Conference Series* (Vol. 1594, Issue 1). Institute of Physics

Publishing. <https://doi.org/10.1088/1742-6596/1594/1/012046>

- Setyawan, D. (2017). Exploration of students' knowledge construction process on geometry through random thinking style and spatial ability of mental rotation level. *Journal of Mathematical Power*, 5(2), 124. <https://doi.org/10.26858/jds.v5i2.3348>
- Silmi Juman, Z. A. M., Mathavan, M., Ambedgara, A. S., & Udagedara, I. G. K. (2022). Difficulties in Learning Geometry Component in Mathematics and Active-Based Learning Methods to Overcome the Difficulties. *Shanlax International Journal of Education*, 10(2), 41–58. <https://doi.org/10.34293/education.v10i2.4299>
- Sinaga, M. E., Harman, H., & Yarmayani, A. (2022). Identification of Misconceptions of Students with Impulsive and Reflective Cognitive Style in Solving Mathematics Problems in Class XII MIPA at SMAN 3 Jambi City. *PHI: Journal of Mathematics Education*, 6(2), 229. <https://doi.org/10.33087/phi.v6i2.217>
- Siregar, S. N., Kartini, K., Roza, Y., & Baroqah, L. (2020). Student Worksheets Using the PISA Context for Statistics Topic in Class VIII SMP. Consider using the Doctrine of Religion (2019), 105–119. <https://doi.org/10.24071/seadr.2019.15>
- Soleh, D. A. (2021). Analyzing Conceptual and Procedural Knowledge of Geometry among Prospective Teachers: Indonesian Perspective. *Journal of Physics: Conference Series*, 1752(1). <https://doi.org/10.1088/1742-6596/1752/1/012067>
- Sutton, J., & Krueger, A. (2009). *EDthoughts : what we know about mathematics teaching and learning* (Issue Mm).
- Swaroop, A., Singh, A., Chandra, G., Prakash, S., Yadav, S. K., Yang, T., & Rathore, R. S. (2024). A Comprehensive Overview of Formal Methods and Deep Learning for Verification and Optimization. 2024 International Conference on Decision Aid Sciences and Applications, DASA 2024. <https://doi.org/10.1109/DASA63652.2024.10836654>
- Thomson, S., Hillman, K., & Lisa de Bortoli. (2013). *A Teacher's Guide to PISA Mathematical Literacy*. https://www.acer.org/files/PISA_Thematic_Report_-_Maths_-_web.pdf
- Turmuzi, M., Suharta, G. P., Astawa, W. P., & Suparta, N. (2024). MISCONCEPTIONS OF MATHEMATICS IN HIGHER EDUCATION UNIVERSITIES WHEN LEARNING WITH GOOGLE CLASSROOM BASED ON LEARNING STYLES AND GENDER DIFFERENCES. *Journal of Technology and Science Education*, 14(1), 200–223. <https://doi.org/10.3926/jotse.2482>
- Utami, C. T. P., Mardiyana, & Triyanto. (2019). Profile of students' mathematical representation ability in solving geometry problems. *IOP Conference Series: Earth and Environmental Science*, 243(1). [24](https://doi.org/10.1088/1755-</p></div><div data-bbox=)

1315/243/1/012123

- van den Heuvel-Panhuizen, M., Elia, I., & Robitzsch, A. (2014). Learning Mathematics with Picture Books. North American Chapter of the International Group for the Psychology of Mathematics Education, 5, 313–320.
- Varma, R., Verma, Y., Vijayvargiya, P., & Churi, P. P. (2021). A systematic survey on deep learning and machine learning approaches of fake news detection in the pre- and post-COVID-19 pandemic. *International Journal of Intelligent Computing and Cybernetics*, 14(4), 617–646. <https://doi.org/10.1108/IJICC-04-2021-0069>
- Weigand, H. G., Trgalova, J., & Tabach, M. (2024). Mathematics teaching, learning, and assessment in the digital age. *ZDM - Mathematics Education*, 56(4), 525–541. <https://doi.org/10.1007/s11858-024-01612-9>
- Welji, S. N., Drimalla, J., Crawford, B. A., Alibek, A., Conner, A., Franco, L., Menke, J., & Foutz, T. (2022). Analyzing Teacher Supports for Collective Argumentation in Integrative STEM Classrooms (RTP). *ASEE Annual Conference and Exposition, Conference Proceedings*. <https://www.scopus.com/inward/record.uri?eid=2-s2.0-85138279250&partnerID=40&md5=3d7dc1c563ad7c760e2d5fbeece10ecca>
- Widada, W., Agriyanto, S., & Octizasari, G. (2018). The Ability of Trigonometry through the Learning Model of Connected Mathematics and the Scientific Approach. Seminar on Advances in Mathematics, Science, and Engineering for Elementary Schools Mercure Hotel Yogyakarta, 16 August 2018 Website: <Http://Samses2018.Upiconf.Org> Email: Samses@upi.Edu, August.
- Widada, W., Herawaty, D., Nugroho, K. U. Z., & Anggoro, A. F. D. (2021). Augmented Reality assisted by GeoGebra 3-D for geometry learning. *Journal of Physics: Conference Series*, 1731(1), 0–9. <https://doi.org/10.1088/1742-6596/1731/1/012034>
- Wittmann, E. C. (2020). *Connecting mathematics and mathematics education*. In Springer. Springer. <https://doi.org/10.1007/9783030615703>
- Xu, F., Wenhui, K., & Pei, J. (2025). Exploring the effect of digitization on education: An empirical analysis based on PISA 2022. *Education and Information Technologies*. <https://doi.org/10.1007/s10639-025-13414-x>
- Zakariya, Y. F. (2023). Approaches to learning mathematics: preliminary evidence of a concise, valid, and reliable instrument. *Frontiers in Psychology*, 14. <https://doi.org/10.3389/fpsyg.2023.1286394>
- Zeng, C. (2024). The effects of students' backgrounds, attitudes, and ICT familiarity on mathematical literacy: latent profile analysis and lasso regression. *Education and Information Technologies*. <https://doi.org/10.1007/s10639-024-13028-9>
- Nugroho, K. U. Z., Sukestiyarno, Y. L., & Nurcahyo, A. (2021). The Weaknesses

of Euclidean Geometry: A Step of Needs Analysis of Non-Euclidean Geometry Learning through an Ethnomathematics Approach. *Edumatika : Jurnal Riset Pendidikan Matematika*, 4(2), 126–149.

<https://doi.org/10.32939/ejrpm.v4i2.1015>

Nugroho, K. U. Z., Widada, W., Herawaty, D., Tuzzahra, R., Panduwinata, B., & Sospolita, N. (2020). Abstraction Ability of Students About Fractions Through Local Cultural Approaches. *Proceedings of the International Conference on Educational Sciences and Teacher Profession (ICETeP 2020)*, 480–485. <https://www.atlantis-press.com/proceedings/icetep-20>

Wahyudi, N., Nugroho, K. U. Z., & Herawaty, D. (2019). Modifikasi Software Lisrel Dengan Membuat Teknik Analisis Konstruksi Validitas Instrumen Tes. *Maret*, 05(01), 82–90. <https://ejournal.unib.ac.id/index.php/jpmr>